Chapter 1 – Review Mathematician ___________________________

- All homework FOR CHAPTERS 2-8 FOR AB AND 2-10 FOR BC will be on MathXLforSchool. You will easily be able to tell when we are getting close to a test by noticing how many sections are in a chapter. The test is always soon after the last section. The exception to this – Chapters 3 and 4 are all on one test.
- You will all get your MathXL license and information on the same day after the Chapter 1 test.
- Whenever I give classwork – or any assignment or test - that is on paper - *YOU MUST SHOW WORK ON ALL THAT IS SUBMITTED*. Most of your points are earned in the work that leads to the final answer on the AP test free response section. You must get accustomed to showing all work.
- In chapter 1 only, I want all of the homework sections handed in at once the day *before* the test. Chapter 1 is the only "regular" homework assignment that will earn points.
- There will be no penalty for not completing MathXL homework. But those who do ALL OF THE HOMEWORK with a minimum of 90% will have 3 percentage points tacked on to the nine weeks grade. (Homework bonus goes on the first 3 nine weeks)
- In chapters 2-10, all MathXL work is expected to be done within two days of the completion of the lecture –*Homework is a bonus!* There is no way to do it after the date because it will no longer be available online.
- If there is something in the homework that you didn't glean from my lecture and the notes – You should look at the posted videos and / or come to see me!!!
- For full credit for submitted work, you must submit college/professional-level work. Use paper that does not have rough edges (This includes classwork, misc extra problems that I give, curves, anything!) that you submit to me.

  I expect to receive college-level work from college-level students.

  Included with that - **I want headings on all of the papers that you give to me. Your name, period, date, and both the section AND the page number. NOTE – Work that is not what I consider to be college-level work with a college-level appearance will not even be considered for grading.**

Section 1

TEST QUESTION –
(As in – this next example – IT WILL BE O N THE TEST WITH DIFFERENT NUMBERS…)
There are four standard forms of the line - name them, show their basic form, and re-write $6y-4=7x$ in these four forms.

Pt-slope	$y-y_1 = m(x - x_1)$
General	$Ax + By + C = 0$ Note that A, B, and C are whole numbers
Slope-Intercept	$y = mx + b$
Intercept	$\frac{x}{a}+\frac{y}{b}=1$

Rewrite the given equation in those four forms

Note: You may not be able to write an equation in all four ways. If the line is vertical, you won't be able to write the equation in three of the forms!

Show the work that leads to the shortest distance from the line 12y+7x= - 41 to the point (8, 8)

Note: There is an equation that will lead you to the answer, but I want to see that you can do the work (You may use the equation as a check) –

$$\text{distance}(ax+by+c=0;(x_0,y_0))=\frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}$$

To find this (the long way using your basic knowledge of the distance equation and perpendicular lines) –

Sketch what I am asking you to find.

Solve for y to find the slope of the line
Find the perpendicular slope of this line
Write the equation that has that slope and goes through the given point
Solve for y
You know have two equations. Solve for x and y
Plug that point and the given problem point into the distance formula.

Example: Solve for x: $|x-3|=7$

Example: Rewrite following using absolute values in the form $|x-a|<b$:

$$2<x<10$$

$$|x-\text{middle}|<(\text{distance to endpoints from middle})$$

Solve $\frac{3}{x-5}=1$
(Algebraically and Graph)

Solve $\frac{3}{x-5}<1$
(Algebraically and Graph)

Solve $\left|\frac{3}{x-5}\right|<1$
(Algebraically and Graph)

1.2 Functions and Graphs

You must be able to find the domain and range with all work shown (For domain, isolate y and determine what x canNOT be. For range, isolate x and determine what y canNOT be…)

Examples: Find the domain and range – analytically (a combination of algebra and common sense) for all three. Remember that when you are asked to find the domain, you are being asked, "What values can we use for x?"

$y = \frac{1}{x-3}$	$y = -\sqrt{-x}$	$y = 1 + \frac{1}{x}$

You can recognize that an equation is a function by looking at the graph by using the vertical line test.

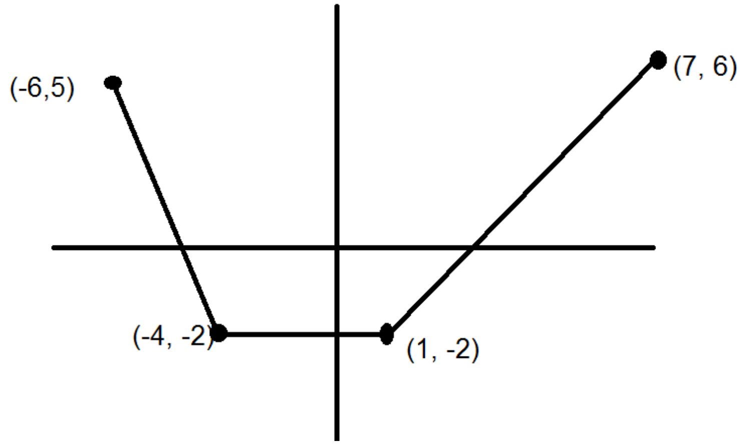
Greatest integer function, aka the floor function, can look like either $\lfloor \text{function} \rfloor$ or $[\text{function}]$
BOTH OF THESE SYMBOLS MEAN THE SAME THING!!!

After you find the value of the equation, if the value isn't a whole number, you round down. For the ceiling function, you round up. The ceiling function will be in this form $\lceil \text{function} \rceil$ (You will rarely see the ceiling function in math problems but should still be able to grasp the concept)

Separate notes attached.

You are expected to find the 0's of an equation on your calculator.

Write this as a piecewise function. First, find the equations of the three given lines.



$$f(x) = \begin{cases} \\ \\ \\ \end{cases}$$

1.3 Even though they don't cover exponentials and logarithms until 1.5, there are some needed in this section:

Properties of logs: In this class, we use NATURAL LOGS – as in log base e.

$e^{\ln x} = x$

$\ln e^{x} = x$

ln(xy) = ln x + ln y

$\ln \frac{x}{y} = \ln x - \ln y$

$\ln x^{y} = y \ln x$

In this class, if you are given a function in terms of log, you will need to get it into terms of natural logs, Use change of base

$\log_x y = \frac{\ln y}{\ln x}$

Find the zeroes of $y = 7 - 5^{x}$

Rewrite the following using base 5; $(25)^{3x}$ = and $\left(\frac{1}{25}\right)^{3x}$ =

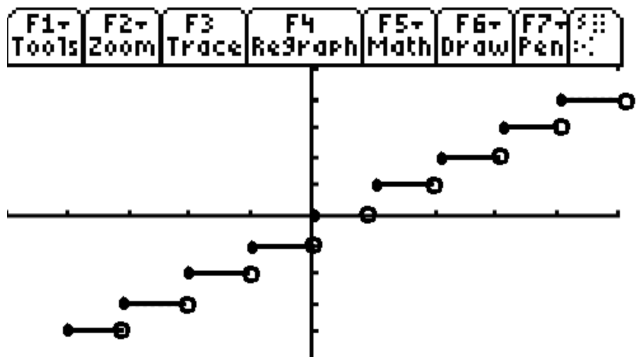
Find the domain and range of $6^{x} - 7 = y$ and $6^{-x} - 7 = y$

Mathematician ___________________________ Work with the greatest integer function

(a) Graph $y = [x] = \lfloor x \rfloor$

x	y before finding gif	Actual y
0	0	0
$\frac{1}{4}$	.25	0
$\frac{1}{2}$	.5	0
1	1	1
1.2	1.2	1
1.8	1.8	1
2	2	2

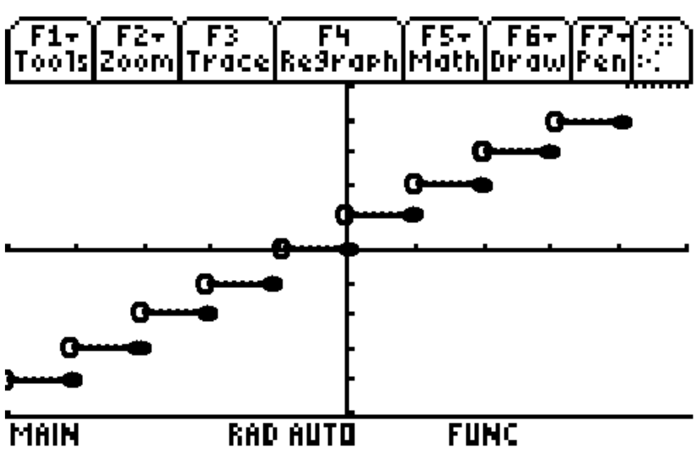
Use a scale of ! on both the x and y axis



(b) Graph $y = \lceil x \rceil$

x	y before finding gif	actual y
0	0	0
$\frac{1}{4}$	.25	1
$\frac{1}{2}$	.5	1
1	1	1
1.2	1.2	2
1.8	1.8	2
2	2	2

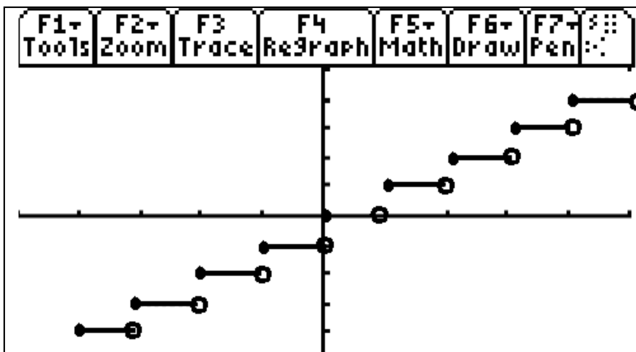
Use a scale of 1 for both the x and y axis.



Note that the floor functions have their “open circles” on the right. The ceiling function has the open circles on the left.
You can use the above graphs for the following:

Example –
(c) Graph $y = [x+1] = \lfloor x+1 \rfloor$

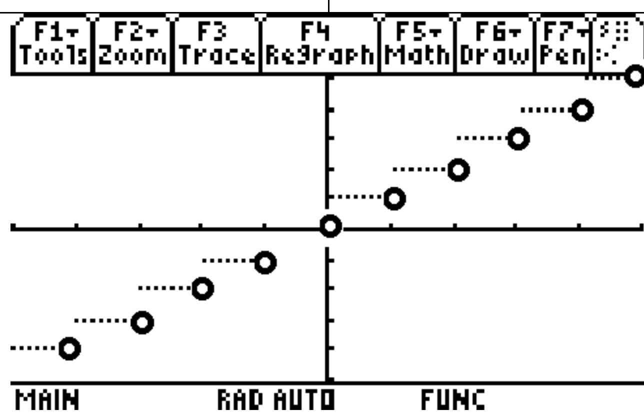
x	y before finding gif	actual y
0	1	1
$\frac{1}{4}$	1.25	1
$\frac{1}{2}$	1.5	1
1	2	2
1.2	2.2	2
1.8	2.8	2
2	3	3



This is the graph from part a…

We’re off by just a little – Think of the origin as point (0,1).

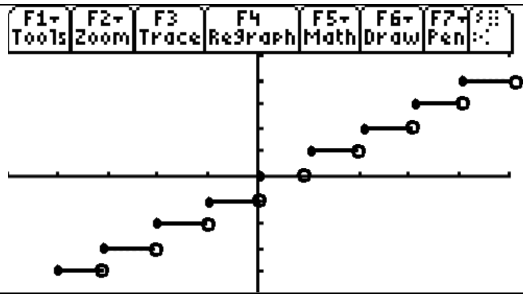
In other words - Move each of the “steps” up one on the y-axis so that it looks like the next graph



What about when we change the coefficient of x?

(d) $y = [2x] = \lfloor 2x \rfloor$. The 2x tells me that at every 1/2-increment, I’m going to have an integer, so I need to check a point between 0 and ½.

x	y before finding gif	actual y
0	0	0
$\frac{1}{4}$	.5	0
$\frac{1}{2}$	1	1
1	2	2



I'm going to use the same graph that I used in (a)

The scale on the x axis is ½

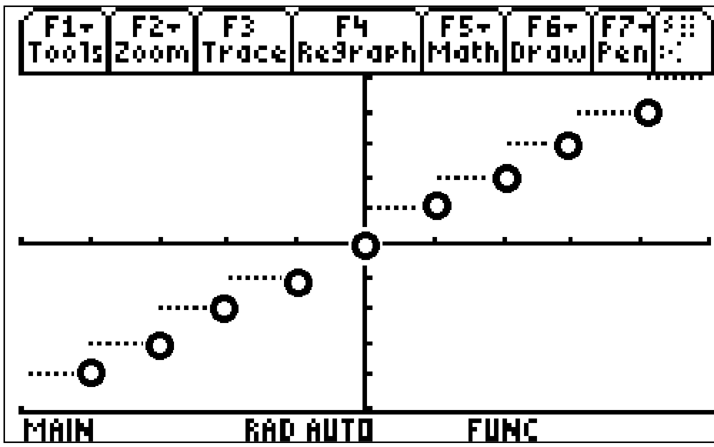
The scale on the y axis is still 1.

(e) With $y = [3x] = \lfloor 3x \rfloor$, picture looks the same. Change the scale along the x-axis to 1/3's.

What about

(f) $y = [3x+1] = \lfloor 3x+1 \rfloor$

x	y before finding gif	actual y
0	1	1
$\frac{1}{6}$	1.5	1
$\frac{1}{3}$	2	2
$\frac{2}{3}$	3	3



Same idea – use the graph from (c) except change the x scale to 1/3's. y scale stays the same.

Try this one…

$y = [2x]+1 = \lfloor 2x \rfloor + 1$

x	2x	[2x]	[2x] + 1

1.4 Parametric Equations

Parametric equations are in the form x = f (t) and y = f (t)

Put your calculator into parametric mode.

Let's look at an example –

Change your window –
0 < t < 20 (t step is .01) -30 < x < 30 -30 < y < 30

x = t cos(t)
y = t sin(t)

This is called Archimedes Spiral

Now –

Graph the following three equations using these conditions –
xmin = 0 xmax = 15 ymin = -5 ymax = 5 tstep = .1

Copy / Sketch the graph in the box – Change the t min and t max for each graph

$x = \frac{3t^2}{4}, \ y = t, \quad -4 \le t \le 4$	$x = 3t^2 + 12t + 12$ $y = 2t + 4, \quad -4 \le t \le 0$	$x = 3t^{\frac{2}{3}}$ $y = 2\sqrt[3]{t}, \quad -8 \le t \le 8$

Eliminating the parameter – Get rid of the t and write the parametric equation in Cartesian format

Identities –

$\cos t = \frac{x}{a}$ and $\sin t = \frac{y}{a}$

(a) Graph the following on your calculator

$x = 6\cos t \qquad y = 6\sin t$

Use $0 \le t \le 2\pi \qquad -20 \le x \le 20, \quad -10 \le y \le 10$ (note: I often use a t-step of .1 – anything smaller starts to get a little long in graphing. I usually use x and y steps of 1)

What is this?

Show this by using properties that you should already know –

In particular for this one $\cos^2 t + \sin^2 t = 1$

Now – put your grapher into trace mode (get the little bouncy ball)

Technically, this graph can go on forever depending on my t values. Change t to stop at 4π. What happened?
How would we change this if we wanted three revolutions? ½ a revolution?

Now try this – Graph and put in Cartesian form
$x = 2\cos t \quad y = 3\sin t; \quad 0 \le t \le 2\pi$

Sometimes, we have to go the other way –

Write $y = x^2 - 4$ in parametric format.

You could have numerous solutions, but you need to check your work!

Let's investigate which ones work and which ones don't – and why?

Change your window

$-4 \le t \le 4$

In parametric mode –

If x = t, what would y equal? Graph this in parametric mode. Does this look like what you know $y = x^2 - 4$ should look like? If not – why?

If $x = t^2$, what would y equal? Graph this in parametric mode. Does this look like what you know $y = x^2 - 4$ should look like? If not – why?

If $x = t^3$, what would y equal? Graph this in parametric mode. Does this look like what you know $y = x^2 - 4$ should look like? If not – why?

Sometimes you will be given two points and asked to parametrize them.

The standard form for this is
$x = x_1 + at$ and $y = y_1 + bt$

(2, -3) and (1, 4). Parametrize the line that would go through these two points.

Note: the beginning t value that you use finds the *initial point*. The ending t value that you use is called the *terminal point*. Plug them into the x and y to find the actual points.

Find the initial and terminal point for

$$x = \frac{3t^2}{4},\ y = t, \quad -4 \le t \le 4$$

If t is starting at -4, then the initial point is (12, -4). The graph "ends" when we get to t=4 which is (12,4)

And for the coolest thing in parametrics… Inverses…

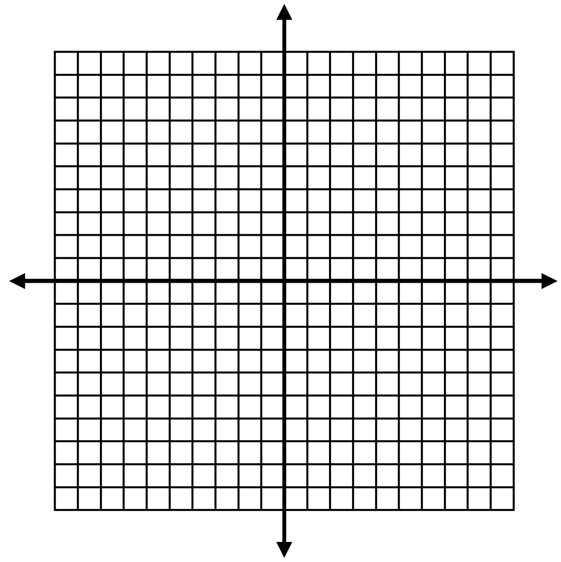
Pre-Calculus Parametrics Worksheet Name ____________________________
Show work on separate paper.

1. Fill in the table, plot the points, and sketch the parametric equation for t [-2,6]

$x = \sqrt{t^2 + 1}$

$y = 2 - t$

t	x	y
-2		
-1		
0		
1		
2		
3		
4		
5		
6		



Problems 2 – 10: Eliminate the parameter to write the parametric equations as a rectangular equation. Check your work by first graphing the parametric equations (on your calculator) than graphing the Cartesian equations (also on your calculator) to see if they match. (Note: If you graph faster by hand – go for it)

2. $x = \frac{1}{t - 2}$ $y = 4t + 5$	3. $x = 6 - t$ $y = \sqrt{3t - 4}$	4. $x = \frac{1}{2}t + 4$ $y = t^3$
5. $x = 3 \cos t$ $y = 3 \sin t$	6. $x = 4 \sin (2t)$ $y = 2 \cos (2t)$	7. $x = \cos t$ $y = 2 \sin^2 t$
8. $x = 4 \sec t$ $y = 3 \tan t$	9. $x = 4 + 2 \cos t$ $y = -1 + 4 \sin t$	10. $x = -4 + 3\tan t$ $y = 7 - 2 \sec t$

For the next two questions: Write two new sets of parametric equations for the following rectangular equations.

11. $y = (x + 2)^3 - 4$

12. $x = \sqrt{y^2 - 3}$

Chapter 1 Section 5

To find the inverse of a function - switch the x's and y's and try to isolate the y.

Inverses in Parametric World are so easy!

Graph $y = x^3 - 7x^2 + 9$; graph the inverse; and graph the line y = x.

Chapter 1 Section 6

You must know the unit circle.

All angles that we use in this class will be in radians - to convert degrees to radians, use the formula
$\frac{\pi}{180} = \frac{x \quad \text{radians}}{y \quad \text{degrees}}$ (and if the book tries to trick you and gives you degrees, you need to convert to radians)

Formulas to remember (that we use often in this class)

$\cos^2 x = \frac{1+\cos 2x}{2}$ $\sin^2 x = \frac{1-\cos 2x}{2}$ $\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

$\sec x = \frac{1}{\cos x}$ $\csc x = \frac{1}{\sin x}$

It is assumed that you know cos θ is an even function and sin θ is an odd function which implies that you remember that cos θ = cos (-θ) and that (- sin θ) = sin (-θ)

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cos (A-B)=cos A cos B + sin A sin B
cos (A+B)=cos A cos B - sin A sin B
In particular, cos (2A) = $\cos^2 A - \sin^2 A$***
sin (A-B) = sin A cos B - cos A sin B
sin (A+B) = sin A cos B + cos A sin B
In particular, sin (2A) = 2 sin A cos A********

$\sin^2\theta+\cos^2\theta=1$
$\sec^2\theta-\tan^2\theta=1$

Law of sines $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Law of cosines $c^2 = a^2 + b^2 - 2ab \bullet \cos C$

The equation for finding arc <u>length</u> is s = r θ

The equation for finding sector <u>area</u> is $\frac{1}{2}r^2\theta$

Know that inverse trig functions have only one answer – and that they are only good for certain quadrants.

In which quadrants will you find the following?

$\cos^{-1} x$

$\sin^{-1} x$

$\tan^{-1} x$

$\sec^{-1} x$

$\csc^{-1} x$

$\cot^{-1} x$

Find the exact value of the expression.

1. $cos^{-1}(-1)$
2. $sin^{-1}(0.5)$
3. $tan^{-1}(\sqrt{3})$
4. $\arctan(-1)$
5. $csc^{-1}(\sqrt{2})$
6. $\arcsin 1$
7. $cot^{-1}(-\sqrt{3})$
8. $sec^{-1}(2)$
9. $\sin(sin^{-1}(0.7)$
10. $cos^{-1}(\sin(1))$
11. $tan^{-1}(tan\frac{4\pi}{3})$
12. $\tan(cos^{-1}(0.5)$
13. $\sin(cos^{-1}(\frac{4}{5})$
14. $\sec(\arctan(2))$
15. $\arcsin(sin\frac{5\pi}{4})$

Fill in ALL points and the radians for all the angles drawn.